SUMMER 1984
MATHEMATICS
ORDINARY LEVEL
SYLLABUS D
PAPER 1
(Two and a half hours)

All questions may be attempted.
Answers are to be written on the question paper in the spaces provided, and the question paper is to be handed in at the end of the examination.
If working is needed for any question, it must be shown in the space below that question.
NEITHER MATHEMATICAL TABLES NOR SLIDE RULES NOR CALCULATORS MAY BE BROUGHT INTO THE EXAMINATION ROOM
Questions 1 to 23 carry 3 marks each;
Questions 24 to 28 carry 5 marks each;
Question 29 carries 6 marks.
NEITHER MATHEMATICAL TABLES NOR SLIDE RULES NOR CALCULATORS MAY BE USED IN THIS PAPER.

1 Given that \( x = 4 \) and \( y = -3 \), evaluate
   (i) \( 2x + 3y \),
   (ii) \( 7 - y^2 \),
   (iii) \( y(x - y) \).

   Answer (i) __________________________
   (ii) __________________________
   (iii) __________________________

2 Calculate the exact value of
   (i) \( 3.1 \times 0.07 \),
   (ii) \( 73.2 \div 0.4 \).

   Answer (i) __________________________
   (ii) __________________________
3 Given that \( x \leq 12 \frac{1}{2} \), state the largest possible value of \( x \) if
   (i) \( x \) is an integer,
   (ii) \( x \) is a prime number,
   (iii) \( x \) is a rational number.

Answer (i) ______________________________________
Answer (ii) ______________________________________
Answer (iii) ______________________________________

4 (i) Factorise completely \( 18x^2y - 30xy^2 \).
(ii) Factorise \( 1 - p - 12p^2 \).

Answer (i) ______________________________________
Answer (ii) ______________________________________

5 Given each answer as a fraction in its lowest terms, find the exact value of
   (i) \( \frac{3}{4} \times \frac{2}{3} \),
   (ii) \( (3\frac{1}{2} - 1\frac{3}{4}) + 3\frac{1}{2} \).

Answer (i) ______________________________________
Answer (ii) ______________________________________

6 (a) Express \( 0.003186 \) correct to 3 significant figures.
(b) Express \( 52.300 \) in standard form.
(c) Express \( \frac{3}{5} \) as a decimal.

Answer (a) ______________________________________
Answer (b) ______________________________________
Answer (c) ______________________________________

7 Evaluate the matrix product
   (i) \( \begin{pmatrix} 2 & 0 & 2 \\ 0 & 4 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} \).
   (ii) \( \begin{pmatrix} 2 & 5 \\ -2 & 3 \end{pmatrix} \begin{pmatrix} 3 \\ -4 \end{pmatrix} \).

Answer (i) ______________________________________
Answer (ii) ______________________________________

8 (a) Calculate the exact value of
   (i) \( (763 - 287) \div 4 \),
   (ii) \( 6 \div 8.7 \).
(b) Given that \( 2^n \times 2 = 2^k \), write down the value of \( k \).

Answer (a) (i) ________________________________
Answer (a) (ii) ________________________________
Answer (b) \( k = \) _____________________________

9 (a) Express without brackets in its simplest form \( (3p - 2)(5p - 4) \).
(b) Solve the equation \( 2x + 3 = 4(x + 1) \).

Answer (a) ______________________________________
Answer (b) ______________________________________

10 For the distribution 5, 8, 12, 10, 5, 3, 7, 5, 20, 10, find
   (i) the mode,
   (ii) the mean,
   (iii) the median.

Answer (i) Mode ________________________________
Answer (ii) Mean ________________________________
Answer (iii) Median ________________________________
11 (a) A lady buys an article marked at £6.60 but, in addition, has to pay 15\(^\circ\) tax. Calculate the total amount that she has to pay.

(b) When a shopkeeper sells a camera for £84, he makes a profit of 12\(^\circ\). Calculate the cost price of the camera.

Answer (a)

(b)

12 Evaluate

(i) \(361\), (ii) \(81\), (iii) \((2)^2\).

Answer (i)

(ii)

(iii)

13 Solve the simultaneous equations

\[3x + 2y = 4,\]
\[x - 3y = 17.\]

Answer \(x = \ldots\)

\(y = \ldots\)

15 (a) Evaluate \(\cos 115^\circ\), using as much of the information below as is necessary.

(b) In triangle \(PQR, \hat{Q} = 90^\circ, \hat{P} = 65^\circ\) and \(PQ = 4\) cm. Calculate \(QR\), using as much of the information below as is necessary.

\[
\begin{array}{c|c|c}
\text{Angle} & 25^\circ & 65^\circ \\
\hline
\sin & 0.4226 & 0.9063 \\
\cos & 0.9063 & 0.4226 \\
\tan & 0.4693 & 2.145 \\
\end{array}
\]

Answer (a) \(\cos 115^\circ = \ldots\)

(b) \(QR = \ldots\) cm

16 (a) Given that \(\frac{2p}{x} = \frac{a}{b}\), find an expression for \(x\) in terms of \(a, b\) and \(p\).

(b) Taking \(x = \frac{5p}{3}\), calculate the radius of the base of a cylinder, given that its volume is 77 cm\(^3\) and its height is 8 cm.

Answer (a) \(x = \ldots\)

(b) \(\ldots\) cm

17 \(P, Q, R\) and \(S\) are the four corners of a rectangular plot marked out on level ground. Given that the bearing of \(Q\) from \(P\) is 020\(^\circ\) and the bearing of \(R\) from \(P\) is 090\(^\circ\), calculate the bearing of

(i) \(P\) from \(Q\),

(ii) \(R\) from \(Q\),

(iii) \(S\) from \(Q\).

Answer (i)

(ii)

(iii)
18 (i) State the inequality which defines the unshaded area.

Answer (i) ____________________________

(ii) State the inequality which defines the unshaded area.

Answer (ii) ____________________________

19 (a) Calculate the size of an exterior angle of a regular polygon with 20 sides.

(b) In the diagram, GP = 1 cm, PH = 2 cm, HQ = 1 cm, QK = 3 cm and \( \angle H = 90^\circ \).

Calculate the area of the quadrilateral GPQK.

Answer (a) ____________________________

(b) ____________________________ cm²

20 On the axes in the answer space, sketch, and label clearly, the graphs of

(i) \( y = x \),

(ii) \( y = 1 - x^2 \).

Answer ____________________________

21 Express as a single fraction in its simplest form

\[
\frac{3}{2x - 1} - \frac{4}{3x + 2}
\]

Answer ____________________________

22 The sides of a triangle are of length 7 cm, 8 cm and 9 cm.

Calculate, as a fraction in its simplest form, the cosine of the angle opposite the 7 cm side.

Answer ____________________________

23 A class of 30 children entered a competition in which the highest possible score was 6.

Their scores are given in the table.

<table>
<thead>
<tr>
<th>Score</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of children</td>
<td>1</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>9</td>
<td>8</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Draw clearly, on the axes in the answer space, a histogram to represent this data.
25 An aircraft flies from a point \( P(75^\circ \text{N}, 20^\circ \text{W}) \) directly over the North Pole to a point \( Q \).

(i) Given that the distance it flies from \( P \) to the North Pole is \( x \) nautical miles, calculate \( x \).

(ii) Given that the further distance it flies from the North Pole to \( Q \) is \( 2x \) nautical miles, find (a) the latitude of \( Q \). (b) the longitude of \( Q \).

(iii) Another aircraft flies due east from \( P \) to a point \( R(75^\circ \text{N}, 30^\circ \text{E}) \). Calculate, in nautical miles, the distance it flies, using as much of the information below as is necessary.

\[
\sin 75^\circ = 0.9659, \quad \cos 75^\circ = 0.2588, \quad \tan 75^\circ = 3.732.
\]

Answer

(i) \( x = \) 

(ii) (a) \( \text{Latitude} = \) 

(b) \( \text{Longitude} = \) 

(iii) \( \) nm

26 Two six-sided dice, one coloured black and one red, are thrown. Giving each answer as a fraction, calculate the probability that:

(i) the score on the red die is 3,

(ii) each die shows a score of 5,

(iii) the score on the black die is either 2 or 4,

(iv) either the score on the black die is 1, or the score on the red die is 6, but not both.

Answer

(i) 

(ii) 

(iii) 

(iv) 

Answer

(i) 

(ii) 

(iii) 

(iv)
27. P and Q are two transformations, P being represented by the matrix \( \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \).

The diagram in the answer space shows an arrow \( \mathbf{A} \) and its image under the transformation Q.

(i) Draw, and label clearly.
   (a) the arrow \( P(\mathbf{A}) \),
   (b) the arrow \( PQ(\mathbf{A}) \).

(ii) Find the \( 2 \times 2 \) matrix which represents the transformation Q.

**Answer (i)**

\[ \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \]

(ii) matrix is \( \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \)

28. The diagram in the answer space shows the origin \( O \) and three points \( A, B \) and \( P \). The position vectors of \( A \) and \( B \) with respect to \( O \) are \( \mathbf{a} \) and \( \mathbf{b} \).

Given that another point \( Q \) is such that

\[ \overrightarrow{OQ} = h \mathbf{a} \quad \text{and} \quad \overrightarrow{QP} = k \mathbf{b}, \]

(i) mark on the diagram, and label clearly, the point \( Q \),

(ii) determine the value of
   (a) \( h \),
   (b) \( k \).

**Answer (i)**

\[ \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \]

(ii) \( \mathbf{h} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \)

(b) \( k = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \)

19. Two towns, \( A \) and \( B \), are 26 km apart. A cyclist leaves \( A \) at 13.00 and rides towards \( B \) at a steady speed of 16 km/h for 1 hour 15 minutes. He then rests until 15.00 and finally continues at a steady speed of \( \nu \) km/h, arriving at \( B \) at 15.30.

(i) Calculate \( \nu \).

(ii) Calculate, in km/h, the average speed for the whole journey.

(iii) \( \nu \) Draw, on the axes in the answer space, the distance-time graph which represents the journey.

\( \nu \) Given also that a lorry leaves \( B \) at 14.30 and travels to \( A \) at a steady speed of 20 km/h, draw, on the same axes, the graph representing this journey.
EXAMINATION PAPERS (ORDINARY LEVEL)

Answer (i) \( \frac{1}{2} \) km/h

(ii) \( \frac{1}{3} \) km/h

(iii) \( \frac{1}{4} \) km/h

Distances from A (km)

Time of day

1. (a) Find the simple interest obtained when £125 is invested at 8\% per annum for a period of 9 months.

(b) A sum of money is divided in the ratio 2:3:7. Given that the largest share is £112, calculate the smallest share.

(c) The area of one face of a cube is 36 cm\(^2\). Find (i) the volume of the cube, (ii) the total length of all its edges.

2. (a) In a comprehensive school all 200 children in the first year study either Physics, or Chemistry, or both Physics and Chemistry.

Given that 80\% study Physics and 30\% study Chemistry, find the number of children who study

(i) both Physics and Chemistry,
(ii) Physics only.

(iii) Given that the determinant of the matrix \( \begin{pmatrix} 1 & -5 \\ 2 & x \end{pmatrix} \) is equal to the determinant of the matrix \( \begin{pmatrix} 5 & x \\ 3 & 8 \end{pmatrix} \), find the value of \( x \).

(ii) Find the inverse of the matrix \( \begin{pmatrix} 4 & -5 \\ 1 & 2 \end{pmatrix} \).

3. The diagram represents a circular cone of vertical height 6 cm standing on a horizontal base of radius 3 cm.

- \( V \) is the vertex of the cone,
- \( O \) is the centre of the base and \( P \) is a point on the circumference of the base.

Calculate

(i) \( V_P \),

(ii) \( V_O \),

(iii) the circumference of the base of the cone, taking \( \pi \) to be 3.142.

A similar cone has a vertical height of 12 cm.

(iv) Write down, in the form 1:n, the ratio of the volume of the smaller cone to that of the larger cone.
Answer the questions in this section. Each question in this section carries twelve marks.

6 A salesman was paid his annual salary in twelve equal monthly instalments. In addition at the end of each year, he was paid a bonus which amounted to 7\% of the value of his total annual sales.

Given that his annual salary was £5100 and that his total sales during the first year amounted to £20 000, calculate

(i) his monthly salary, \[ \text{[1]} \]
(ii) his total income in the first year. \[ \text{[2]} \]

During the second year his annual salary remained unchanged but his total income for the year amounted to £34 540.

(iii) Calculate his total sales during the second year. \[ \text{[3]} \]

In the third year his annual salary was increased to £58 650 and his bonus was increased to 8\% of the value of his total annual sales. Calculate

(iv) the percentage increase in his annual salary, \[ \text{[2]} \]
(v) the sales, correct to the nearest £1000, he had to achieve in the third year, if his total income during the year was to be at least £13 000. \[ \text{[3]} \]

7 The angles \( \angle A, \angle B \) and \( \angle C \) of a triangle \( ABC \) are 45\%, 54\% and 81\% respectively. \( BC \), the shortest side of the triangle, is 12 cm long. Calculate

(i) the length of the longest side of the triangle, \[ \text{[4]} \]
(ii) the length of the shortest perpendicular height of the triangle. \[ \text{[2]} \]

The perpendicular bisector of \( BC \) meets \( BC \) at \( M \) and \( BA \) at \( N \). Calculate

(iii) \( MN \), \[ \text{[3]} \]
(iv) \( CN \). \[ \text{[3]} \]

8 (a) \[ \text{[5]} \]

5 The equation of a straight line is \( 3y + 2x + 6 = 0 \).

Calculate

(i) the gradient of the line, \[ \text{[1]} \]
(ii) the coordinates of the point where the line crosses the y-axis, \[ \text{[2]} \]
(iii) the coordinates of the point at which the line intersects the line \( y = 4 \), \[ \text{[2]} \]
(iv) the equation of the line, parallel to the given line, which passes through the point (5, 2). \[ \text{[3]} \]

\( KLMN \) is a trapezium in which \( KL \) is parallel to \( NM \) and \( \angle KLM = 90\% \).

(i) Given that \( KL = (3x - 1) \text{ cm}, \) \( NM = (x + 3) \text{ cm} \) and \( LM = (x - 3) \text{ cm} \), find, in terms of \( x \), an expression for the area of the trapezium.

(ii) Given also that the area of the trapezium is 15 cm\(^2\), form an equation in \( x \) and show that it reduces to

\[ 2x^2 - 5x - 18 = 0. \]

(iii) Solve this equation and hence find the length of \( LM \). \[ \text{[8]} \]
EXAMINATION PAPERS (ORDINARY LEVEL)

(b) Given that \( p = 2t + 1 \) and \( q = t^2 + 1 \),
(i) find the values of \( p \) and \( q \) when \( t = 5 \).
(ii) express \( \frac{p}{2q} \) in terms of \( t \), giving your answer in its simplest form. [4]

\[ \text{Diagram of a rectangle with mid-point O.} \]

\( ABCD \) is a rectangle and \( O \) is the mid-point of \( AD \).
A semicircle of radius 7 cm is drawn on \( AD \) as diameter. The semicircle cuts the side \( BC \) at \( P \) and \( R \) such that \( \angle POA = 30^\circ \). Calculate
(i) \( AD \), [1]
(ii) \( AB \), [2]
(iii) \( BP \). [3]

Taking \( \pi \) to be \( \frac{22}{7} \), calculate
(iv) the length of the arc \( PQR \), [3]
(v) the area of the shaded segment \( PQR \). [4]

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The triangle \( PQR \) with vertices \( P(3, 2) \), \( Q(1, 1) \) and \( R(3, 6) \) is shown in the diagram.
(i) An enlargement maps \( \triangle PQR \) onto \( \triangle PAB \). Given that the coordinates of \( A \) are \((m, 0)\),
(a) the centre of the enlargement,
(b) the value of \( m \),
(c) the scale factor of the enlargement,
(d) the coordinates of the point \( B \),
(e) the ratio of the area of \( \triangle PAB \) to that of \( \triangle PQR \). [5]

(ii) \( C \) is the point \((2, 5)\) and \( D \) is the point \((6, 3)\). Given that a single transformation maps \( \triangle PQR \) onto \( \triangle CQD \),
(a) describe the transformation fully,
(b) write down the matrix which represents this transformation. [4]

(iii) Given that \( PQRS \) is a parallelogram,
(a) write down the coordinates of the point \( S \),
(b) describe fully the single transformation which will map \( \triangle PQR \) onto \( \triangle RSP \). [3]

11 (a)

The diagram is the cumulative frequency curve for the marks of 400 candidates in an examination. Use the curve to estimate, as accurately as possible,
(i) the median mark,
(ii) the inter-quartile range,
(iii) the pass mark, given that 70% of the candidates passed the examination,
(iv) the probability that a candidate scored 80% or less. [7]
EXAMINATION PAPERS (ORDINARY LEVEL)

(b) A bag contains four counters, one marked with the letter A, one with the letter B and two with the letter L.

The counters are drawn at random from the bag, one at a time, without replacement.

In each of the following cases calculate the probability that
(i) the first two counters to be drawn out will each have the letter L marked on them,
(ii) the second counter to be drawn out will be that with the letter B marked on it,
(iii) the order in which the counters are drawn will spell out the word B A L L. [5]

12 Answer the whole of this question on a sheet of plain paper.

(a) (i) Construct a triangle $ABC$ in which $AB = 9 \text{ cm}$, $BC = 7 \text{ cm}$ and $AB = 38^\circ$. Measure, and write down, the length of $AC$.

(ii) Find, and mark clearly with the letter $P$, the two points which are 6 cm from $B$ and equidistant from $AC$ and $AB$.

(iii) The point $Q$, which lies inside the triangle $ABC$, is such that its distance from $B$ is less than 6 cm and it is nearer to $AC$ than to $AB$.

Indicate clearly, by shading, the region in which $Q$ lies.

(b)

In the diagram, $PQRS$ is a straight line and $PQ = QR = RS$.

Given that $\overline{OQ} = 2x + y$ and $\overline{PO} = y - x$, express, as simply as possible, in terms of $x$ and/or $y$,
(i) $\overline{RO}$,
(ii) $\overline{OR}$,
(iii) $\overline{OS}$. [5]

13 Answer the whole of this question on a sheet of graph paper.

A farmer who intended to keep sheep and cows on his farm asked each of his four sons how many sheep and/or cows he should keep.

Alan suggested that there should be more than 10 cows.

Brian suggested that the number of sheep should be at least 20 but not more than 50.

Charles suggested that the total number of sheep and cows should be less than 70.

David suggested that the number of sheep should be greater than or equal to the number of cows.

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Taking $s$ to be the number of sheep and $c$ to be the number of cows, write down the inequalities which represent these conditions.

The point $(s, c)$ represents $s$ sheep and $c$ cows. Using a scale of 2 cm to represent 10 sheep on the horizontal axis and a scale of 2 cm to represent 10 cows on the vertical axis construct, and indicate clearly by shading the unwanted regions, the region in which $(s, c)$ must lie.

Assuming the farmer took all his sons' suggestions into account and that, when he came to sell the animals, he made a profit of £50 on each sheep and £100 on each cow, find the minimum number of cows he kept on his farm to ensure a profit of at least £4 000. [2]

ENVIRONMENTAL SCIENCE

ORDINARY LEVEL

Paper 1 Physical Sciences

(Two and a half hours)

In Part I answer any ten questions, and in Part II answer five questions, including at least one question from each of Sections A and B.

Before handing in, put Part I on top of your answers to Part II and tie both parts firmly but not too closely together.

Names, not symbols, should be used in descriptive work for all reacting chemicals and for the products formed.

Illustrative drawings or diagrams should be given where they are helpful.

All working must be shown.

Figures in squared brackets [ ] indicate approximate mark values.

[The weight of one kg may be taken as 10 N]

Part I [20 marks]

Answer ten questions.

1 An object of volume 3 000 cm$^3$ ($3 \times 10^{-3}$ m$^3$) has a weight of 60 N.

(a) What is the mass of the object? 

(b) What is the density of the material of the object? 

(c) What would be the density of 1 500 cm$^3$ of the same material?